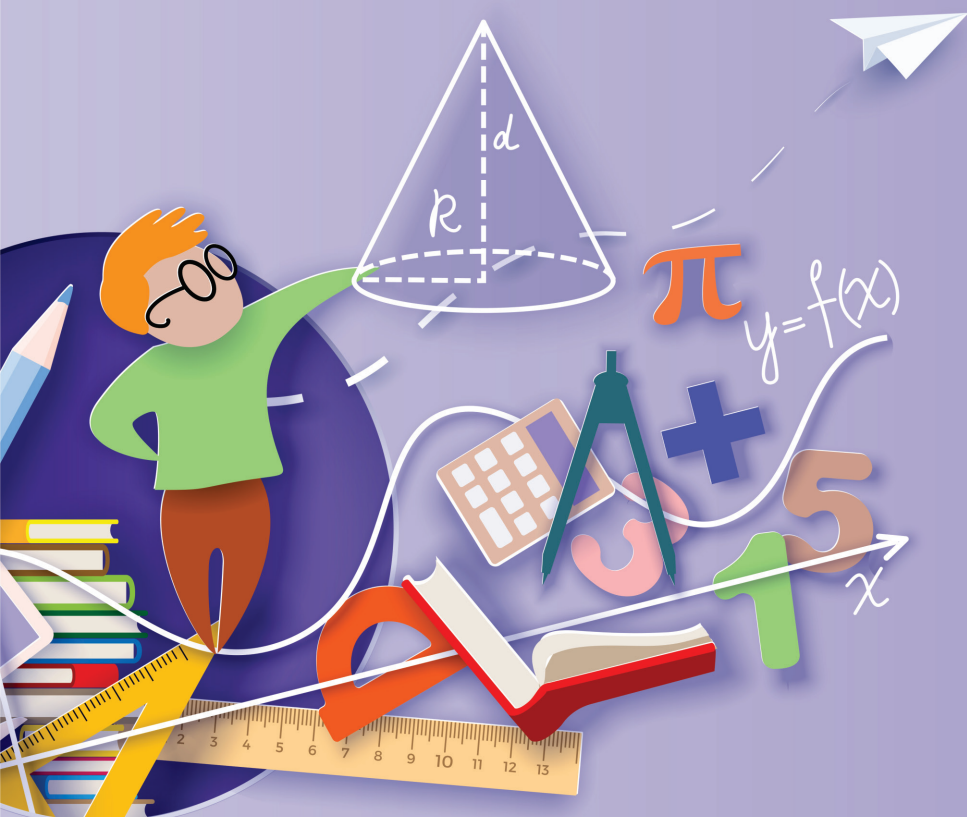




SSC
WALLAH

Quantitative Aptitude Formulae Book

FOR SSC, RAILWAY AND OTHER COMPETITIVE EXAMS



**Quick Revision of Quantitative Aptitude Formulae for
SSC, Railway and Other Competitive Exams**

1. If the prime factorization of a natural number N is

Then, the number of factors of $= (1 + q_1)(1 + q_2)(1 + q_3) \dots$

- 2.** If the prime factorization of a natural number N is given by

Then, the sum of all the factors of N

$$= (1 + p_1^1 + p_1^2 + \dots + p_1^{q_1}) (1 + p_2^1 + p_2^2 + \dots + p_2^{q_2}) \dots$$

3. The factorial of a natural number is equal to the product of the first n natural numbers and it is denoted by $N!$

$$N! = 1 \times 2 \times 3 \times \dots \times N$$

4. Let p be a prime number. Then, the maximum value of m for which p^m

$$\text{divides } N! \text{ completely} = \left\lfloor \frac{N}{p} \right\rfloor + \left\lfloor \frac{N}{p^2} \right\rfloor + \left\lfloor \frac{N}{p^3} \right\rfloor + \dots$$

where $[x]$ denotes the greatest integer less than equal to x .

- 5.** $\text{HCF} \times \text{LCM} = \text{Product of two numbers}$

Note: This formula is applicable only for two numbers

6. If $P = n(n+1)(n+2)(n+3) + 1$, and P is a perfect square number then $P = (n^2 + 3n + 1)^2$

7. A number y to be added to $x(x+a)(x+2a)(x+3a)$ to make it a perfect square then $y = a^4$

8. $\sqrt{a\sqrt{a\sqrt{a\ldots\infty}}} = a$

$$9. \sqrt[n]{a \sqrt[n]{a \sqrt[n]{a \dots (n \text{ times})}}} = a^{\frac{2^n - 1}{2^n}}$$

$$10. \sqrt[3]{a \sqrt[3]{a \sqrt[3]{a \dots \infty}}} = \sqrt{a}$$

$$11. \sqrt{a \div \sqrt{a \div \sqrt{a \div \sqrt{a \dots \infty}}}} = \sqrt[3]{a}$$

$$12. \sqrt{a + \sqrt{a + \sqrt{a + \sqrt{a \dots \infty}}}} = \frac{\sqrt{4a+1}+1}{2}$$

$$13. \sqrt{a - \sqrt{a - \sqrt{a - \sqrt{a \dots \infty}}}} = \frac{\sqrt{4a+1}-1}{2}$$

$$14. \text{ If } X = \sqrt{a + \sqrt{a + \sqrt{a + \sqrt{a \dots \infty}}}}$$

$$\text{and } Y = \sqrt{a - \sqrt{a - \sqrt{a - \sqrt{a \dots \infty}}}}$$

Then, $X - Y = 1$ and $XY = a$

$$15. \sqrt{a + b \sqrt{a + b \sqrt{a + b \sqrt{a + b \sqrt{\dots}}}}} = \frac{\sqrt{4a+b^2}+b}{2}$$

$$16. \sqrt{a - b \sqrt{a - b \sqrt{a - b \sqrt{a - b \sqrt{\dots}}}}} = \frac{\sqrt{4a+b^2}-b}{2}$$

$$17. \text{ If } X = \sqrt{a + b \sqrt{a + b \sqrt{a + b \sqrt{a + b \sqrt{\dots}}}}}$$

$$\text{and } Y = \sqrt{a - b \sqrt{a - b \sqrt{a - b \sqrt{a - b \sqrt{\dots}}}}}$$

Then, $X - Y = b$ and $XY = a$

$$18. \sqrt[w]{\sqrt[x]{\sqrt[y]{\sqrt[z]{a}}}} = a^{\frac{1}{w \times x \times y \times z}}$$

Sequences and Series

1. The sum of the first n natural numbers $= 1 + 2 + 3 + \dots + n = \frac{n(n+1)}{2}$
2. The sum of the first n even natural numbers $= 2 + 4 + 6 + \dots + 2n = n(n+1)$
3. The sum of the first n odd natural numbers $= 1 + 3 + 5 + \dots + 2n-1 = n^2$
4. The sum of squares of the first n natural numbers $= 1^2 + 2^2 + 3^2 + \dots + n^2$
 $= \frac{n(n+1)(2n+1)}{6}$
5. The sum of squares of the first n even natural numbers
 $= 2^2 + 4^2 + 6^2 + \dots + (2n)^2 = \frac{2n(n+1)(2n+1)}{3}$
6. The sum of squares of the first n odd natural numbers
 $= 1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{n(2n-1)(2n+1)}{3}$
7. The sum of cubes of the first n natural numbers
 $= 1^3 + 2^3 + 3^3 + \dots + n^3 = \left[\frac{n(n+1)}{2} \right]^2$
8. The sum of cubes of the first n even natural numbers
 $= 2^3 + 4^3 + 6^3 + \dots + (2n)^3 = 2[n(n+1)]^2$
9. The sum of cubes of the first n odd natural numbers
 $= 1^3 + 3^3 + 5^3 + \dots + (2n-1)^3 = n^2(2n^2-1)$
10. Let a be the first term and d be the common difference of an Arithmetic Progression (AP). Then, the n^{th} term of this AP is given by

$$T_n = a + (n-1)d$$

- 11.** Let a be the first term and d be the common difference of an Arithmetic Progression (AP). Then, the sum of the first n terms of this AP is given by

$$S_n = \frac{n}{2} [2a + (n-1)d]$$

Or

$$S_n = \frac{n}{2} [a + l], \text{ where } l \text{ is the last term of the given AP.}$$

- 12.** Let a be the first term and r be the common ratio of a Geometric Progression (GP). Then, the n^{th} term of this GP is given by

$$T_n = ar^{n-1}$$

- 13.** Let a be the first term and r be the common ratio of a Geometric Progression (GP). Then, the sum of the first n terms of this GP is given by

$$S_n = \frac{a(r^n - 1)}{r - 1},$$

- 14.** Let a be the first term and r be the common ratio of a Geometric Progression (GP). If $|r| < 1$, then the sum of infinite terms of this GP is given by

$$S_\infty = \frac{a}{1-r}$$

- 15.** Arithmetic mean (AM) of two numbers a and $b = \frac{a+b}{2}$

- 16.** Arithmetic mean (AM) of n numbers

$$a_1, a_2, a_3, \dots, a_n = \frac{a_1 + a_2 + a_3 + \dots + a_n}{n}$$

- 17.** Geometric mean (GM) of two numbers a and $b = \sqrt{ab}$

- 18.** Geometric mean (GM) of n numbers $a_1, a_2, a_3, \dots, a_n = (a_1 \cdot a_2 \cdot a_3 \dots a_n)^{\frac{1}{n}}$

- 19.** For n positive numbers, $AM \geq GM$

- 20.** For n negative numbers, $AM \leq GM$

1. Formulae and Identities

- i. $a^2 - b^2 = (a+b)(a-b)$
- ii. $(a+b)^2 = a^2 + b^2 + 2ab$
- iii. $(a-b)^2 = a^2 + b^2 - 2ab$
- iv. $(a+b)^2 + (a-b)^2 = 2(a^2 + b^2)$
- v. $(a+b)^2 - (a-b)^2 = 4ab$
- vi. $(a+b+c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ca$
- vii. $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$
- viii. $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$
- ix. $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$
- x. $a^3 + b^3 = (a+b)^3 - 3ab(a+b)$
- xi. $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$
- xii. $a^3 - b^3 = (a-b)^3 + 3ab(a-b)$
- xiii. $(a+b)^3 + (a-b)^3 = 2a^3 + 6ab^2$
- xiv. $(a+b)^3 - (a-b)^3 = 2b^3 + 6ba^2$
- xv. $(a+b+c)^3 = a^3 + b^3 + c^3 + 3(a+b)(b+c)(c+a)$
- xvi. $a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$$\text{xvii. } a^3 + b^3 + c^3 - 3abc = \frac{1}{2}(a+b+c) \left[(a-b)^2 + (b-c)^2 + (c-a)^2 \right]$$

$$\text{xviii. If } a + b + c = 0, \text{ then } a^3 + b^3 + c^3 = 3abc$$

$$\text{xix. } (a+b+c)^3 = a^3 + b^3 + c^3 + 3[ab(a+b) + bc(b+c) + ca(c+a) + 2abc]$$

$$\text{xx. } (a+b+c)^3 = a^3 + b^3 + c^3 + 3a^2(b+c) + 3b^2(c+a) + 3c^2(b+a) + 6abc$$

2. Factors of some Algebraic expression:

- i. $1 + a + b + ab = (1+a)(1+b)$
- ii. $x + xa + xb + ab = (x+a)(x+b)$
- iii. $1 + a + b + c + ab + bc + ca + abc = (1+a)(1+b)(1+c)$
- iv. $a^4 + a^2 + 1 = (a^2 + a + 1)(a^2 + 1 - a)$
- v. $a^4 + b^4 + a^2b^2 = (a^2 + b^2 + ab)(a^2 + b^2 - ab)$
- vi. $a^4 + b^4 = (a^2 + b^2 + \sqrt{2}ab)(a^2 + b^2 - \sqrt{2}ab)$
- vii. $a^4 + b^4 - 3a^2b^2 = (a^2 - b^2 + ab)(a^2 - b^2 - ab)$

3. Formulae and Identities: $\left(x + \frac{1}{x}\right)$ form

- i. $\left(x + \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} + 2$
- ii. $\left(x - \frac{1}{x}\right)^2 = x^2 + \frac{1}{x^2} - 2$
- iii. If $x^2 + \frac{1}{x^2} = k$, then $x + \frac{1}{x} = \sqrt{k+2}$ and $x - \frac{1}{x} = \sqrt{k-2}$
- iv. If $x + \frac{1}{x} = k$, then $x^2 + \frac{1}{x^2} = k^2 - 2$
- v. If $x - \frac{1}{x} = k$, then $x^2 + \frac{1}{x^2} = k^2 + 2$
- vi. If $x + \frac{1}{x} = k$, then $x - \frac{1}{x} = \sqrt{k^2 - 4}$
- vii. If $x - \frac{1}{x} = k$, then $x + \frac{1}{x} = \sqrt{k^2 + 4}$
- viii. $\left(x + \frac{1}{x}\right)^3 = x^3 + \frac{1}{x^3} + 3\left(x + \frac{1}{x}\right)$

$$\text{ix. } \left(x - \frac{1}{x}\right)^3 = x^3 - \frac{1}{x^3} - 3\left(x - \frac{1}{x}\right)$$

4. Some Other Important formulae

- i. If $x^2 + x + 1 = 0$, then $x^3 = 1$
- ii. If $x^2 - x + 1 = 0$, then $x^3 = -1$
- iii. If $x^2 + \sqrt{3}x + 1 = 0$ or $x^2 - \sqrt{3}x + 1 = 0$, then $x^6 = -1$

5. Remainder Theorem: Let $p(x)$ be a polynomial of degree greater than equal to 1. If $p(x)$ is divided by $(x - a)$, then the remainder $= p(a)$.

6. Factor Theorem: Let $p(x)$ be a polynomial of degree greater than equal to 1. Then, $(x - a)$ is a factor of $p(x)$ if and only if $p(a) = 0$.

7. Relation between zeros and coefficient of a polynomial:

- i. If α and β are the zeros of the quadratic polynomial $ax^2 + bx + c$ then,

$$\alpha + \beta = -\frac{b}{a} \quad [\text{Sum of the roots}]$$

$$\alpha\beta = \frac{c}{a} \quad [\text{Product of the roots}]$$

- ii. If α , β and γ are the zeros of the cubic polynomial, then $ax^3 + bx^2 + cx + d$ then,

$$\alpha + \beta + \gamma = -\frac{b}{a} \quad [\text{Sum of the roots}]$$

$$\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a} \quad [\text{Sum of the roots taken two at a time}]$$

$$\alpha\beta\gamma = -\frac{d}{a} \quad [\text{Product of roots}]$$

8. Quadratic Formula: The roots of the quadratic equation $ax^2 + bx + c = 0$ are:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

9. Discriminant of a Quadratic equation: The discriminant of the quadratic equation $ax^2 + bx + c = 0$ is:

$$D = b^2 - 4ac$$

10. Nature of Roots: The roots of the quadratic equation $ax^2 + bx + c = 0$ are:

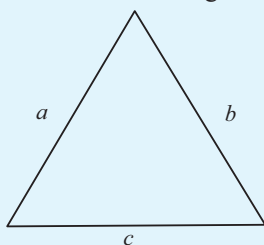
- i. Real and Unequal: If $D > 0$
- ii. Real and Equal: If $D = 0$
- iii. Imaginary: If $D < 0$



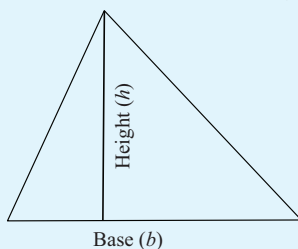
Mensuration Perimetre

1. Triangle:

- i. Let the length of the sides of a triangle be a , b , and c . Then,

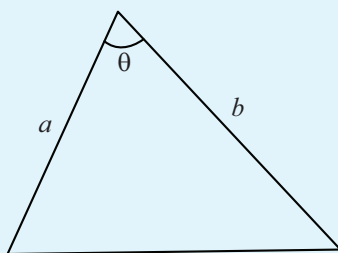


- a. Perimetre of the triangle $= a + b + c$
- b. Semi-perimetre, $s = \frac{a + b + c}{2}$
- c. Area of the triangle, $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$
- ii. Let the base of a triangle be b and let the length of the altitude from the opposite vertex to the base be h . Then,



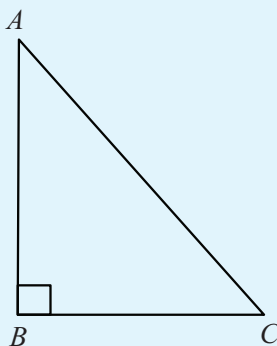
$$\text{Area of the triangle, } \Delta = \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times b \times h$$

- iii. Let a and b be the length of the two sides of a triangle and let θ be the angle between the two lines. Then,



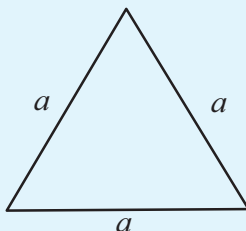
Area of the triangle, $\Delta = \frac{1}{2} \times a \times b \times \sin \theta$

- iv. **Right Angle Triangle:** Let ABC be a right triangle, right angled at B . Then,



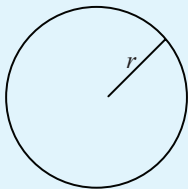
Area of $\Delta ABC = \frac{1}{2} \times \text{base} \times \text{perpendicular} = \frac{1}{2} \times AB \times BC$

- v. **Equilateral Triangle:** Let the length of each side of an equilateral triangle be a . Then,

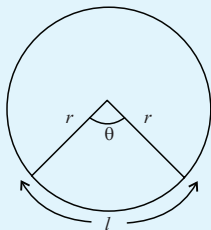


- a. Perimeter of Equilateral Triangle = $3a$
 b. Area of Equilateral Triangle = $\frac{\sqrt{3}}{4} a^2$

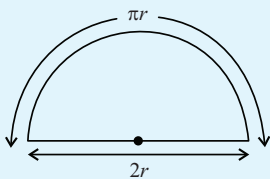
2. Circle: Let the radius of the circle be r . Then,



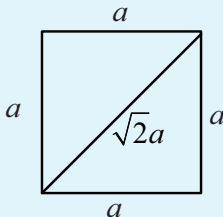
- i. Diameter of the circle, $d = 2r$
- ii. Circumference of the circle $= 2\pi r$
- iii. Area of the circle $= \pi r^2$
- iv. Length of the arc which subtends an angle θ at the centre of the circle
 $l = r\theta$, if θ is in radian
 $l = 2\pi r \times \frac{\theta}{360^\circ}$, if θ is in degree



- v. Area of the sector of central angle $\theta = \pi r^2 \times \frac{\theta}{360^\circ}$
- vi. Perimetre of semi-circle $= \pi r + 2r = r(2 + \pi)$

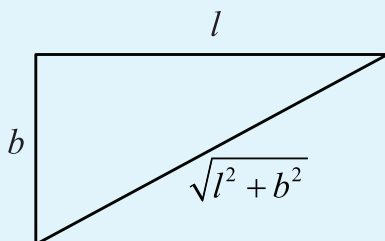


3. Square: Let the length of the side of a square be a . Then,

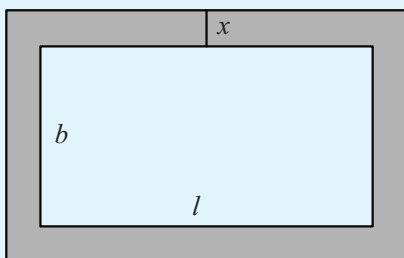


- i. Perimeter of the square = $4a$
- ii. Area of the square = a^2
- iii. Diagonal of the square = $\sqrt{2}a$

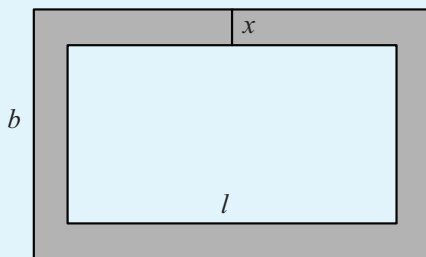
4. Rectangle:



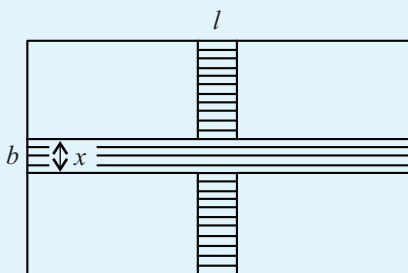
- i. Perimetre of the rectangle = $2(l + b)$
- ii. Area of the rectangle = $l \times b$
- iii. Diagonal of the rectangle = $\sqrt{l^2 + b^2}$
- iv. Area of the path of uniform width x on the outer side along the boundary of the rectangle = $2x(l + b + x)$



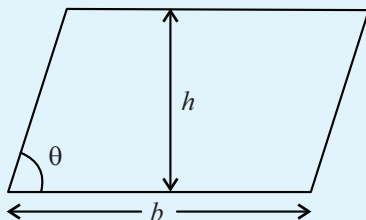
- v. Area of the path of uniform width x on the inner side along the boundary of the rectangle = $2x(l + b - x)$



- vi. Area of the path of uniform width x in the middle of the rectangle along its length and breadth = $x(l + b - x)$



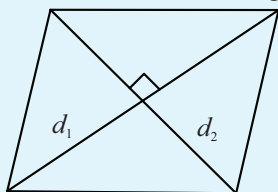
5. Parallelogram:



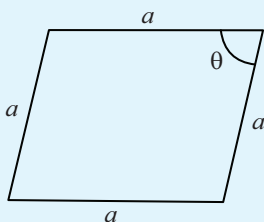
- i. Area of the parallelogram of base b and height $h = \text{base} \times \text{height} = b \times h$
- ii. Area of the parallelogram of the length of the sides a and b and one interior angle $\theta = ab \sin \theta$

6. Rhombus:

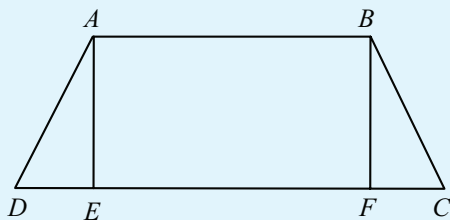
- i. Area of the rhombus of diagonals d_1 and $d_2 = \frac{1}{2} \times d_1 \times d_2$



- ii. Area of the rhombus of the length of the side a and one interior angle $\theta = a^2 \sin \theta$

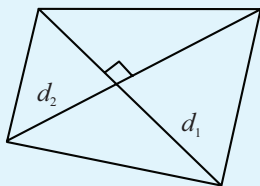


- 7. Trapezium:** Let ABCD be a trapezium with $AB \parallel CD$ and let $AE \perp DC$ and $BF \perp DC$. Then,



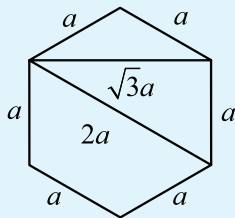
$$\text{Area of the trapezium} = \frac{1}{2} \times (AB + CD) \times AE$$

8. **Quadrilateral:** Let the diagonals of the quadrilateral are d_1 and d_2 and intersect each other at 90° . Then,



$$\text{Area of the quadrilateral} = \frac{1}{2} \times d_1 \times d_2$$

9. **Regular Hexagon:** Let the length of the side of a regular hexagon be a . Then,

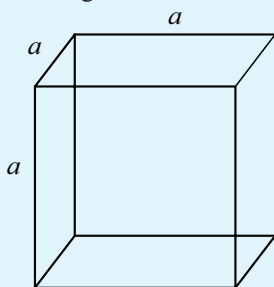


- i. Perimetre of the Regular Hexagon $= 6a$
- ii. Area of the Regular $= \frac{3\sqrt{3}a^2}{2}$
- iii. Length of the longest Diagonal $= 2a$
- iv. Length of the smaller Diagonal $= \sqrt{3}a$

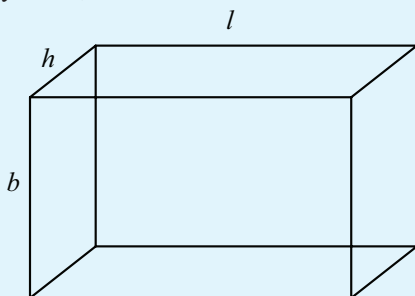
Mensuration

(Volume and surface areas)

1. **Cube:** Let the edge of the cube be a . Then,



- i. Volume of the cube $= a^3$
 - ii. Lateral surface area of the cube $= 4a^2$
 - iii. Total surface area of the cube $= 6a^2$
 - iv. Length of the longest diagonal of the cube $= \sqrt{3}a$
2. **Cuboid:** Let the length, breadth and height of a cuboid be l , b and h respectively. Then,



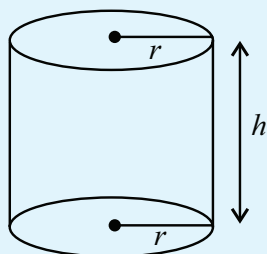
- i. Volume of the cuboid $= l \times b \times h$

ii. Lateral surface area of the cuboid $= 2(l+b) \times h$

iii. Total surface area of the cuboid $= 2(lb + bh + hl)$

iv. Length of the longest diagonal of the cuboid $= \sqrt{l^2 + b^2 + h^2}$

3. **Cylinder:** Let the radius of base and height of the cylinder be r and h respectively. Then,

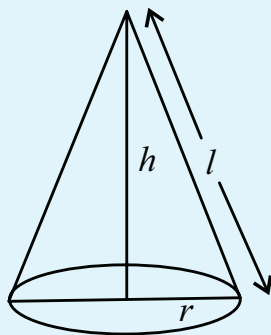


i. Volume of the cylinder $= \pi r^2 h$

ii. Curved Surface Area of the cylinder $= 2\pi r h$

iii. Total Surface Area of the cylinder $= 2\pi r h + 2\pi r^2 = 2\pi r(h + r)$

4. **Cone:** Let the radius of base and height of the cone be r and h respectively. Then,



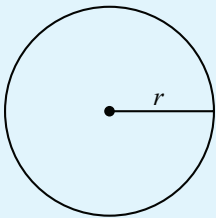
i. Volume of the cone $= \frac{1}{3} \pi r^2 h$

ii. Slant Height of the cone, $l = \sqrt{r^2 + h^2}$

iii. Curved Surface Area of the cone $= \pi r l$

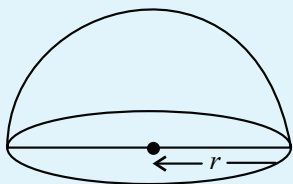
iv. Total Surface Area of the cone $= \pi r l + \pi r^2 = \pi r(l + r)$

5. **Sphere:** Let the radius of the sphere be r . Then,



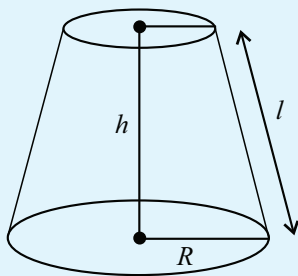
- i. Volume of the sphere $= \frac{4}{3}\pi r^3$
- ii. Total Surface Area of the sphere $= 4\pi r^2$

6. **Hemisphere:** Let the radius of the hemisphere be r . Then,



- i. Volume of the hemisphere $= \frac{2}{3}\pi r^3$
- ii. Curved Surface Area of the hemisphere $= \pi r^2$
- iii. Total Surface Area of the hemisphere $= 3\pi r^2$

7. **Frustum of Cone:** Let the radii of the frustum be r and R and the height of the frustum be h . Then,

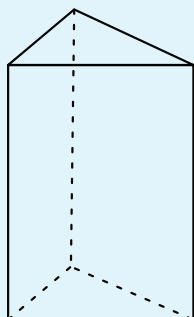


- i. Volume of the frustum of the cone $= \frac{1}{3}\pi h(R^2 + r^2 + Rr)$
- ii. Slant Height of the frustum of cone, $l = \sqrt{h^2 + (R - r)^2}$

iii. Curved Surface Area of the frustum of cone $= \pi l(R + r)$

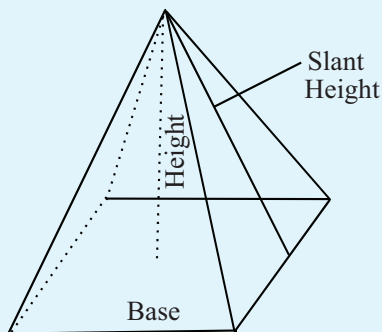
iv. Total Surface Area of the frustum of cone $= \pi l(R + r) + \pi R^2 + \pi r^2$

8. Prism: A prism is a solid whose cross-sectional area is uniform.



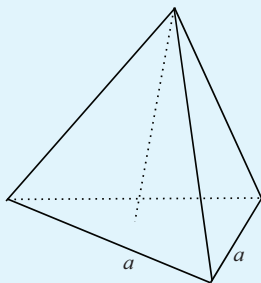
- i. Volume of the prism = Area of the base $\times$ height of the prism
- ii. The curved surface area of the prism = Perimetre of the base $\times$ height of the prism
- iii. The total surface area of the prism = Curved Surface Area $+ 2 \times$ Area of the base.

9. Pyramid:



- i. Volume of the pyramid $= \frac{1}{3} \times$ Area of the base $\times$ Height of the pyramid
- ii. Lateral surface area of the pyramid
 $= \frac{1}{2} \times$ Perimetre of the base $\times$ Slant height of the pyramid
- iii. Total surface area of the pyramid
 $=$ Curved Surface Area $+ \text{Area of the base}$

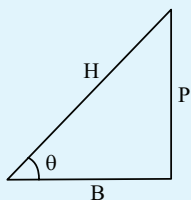
- 10. Regular Tetrahedron:** A regular tetrahedron is a tetrahedron whose all four faces are equilateral triangles of equal size. Let the side of each equilateral triangle be a . Then,



- i. The volume of the regular tetrahedron $= \frac{\sqrt{2}a^3}{12}$
- ii. Lateral Surface Area $= \frac{3\sqrt{3}}{4}a^2$
- iii. Total Surface Area $= \sqrt{3}a^2$

Trigonometry

1. Trigonometric Ratios:



$$\text{i. } \sin \theta = \frac{\text{perpendicular}}{\text{hypotenuse}} = \frac{P}{H}$$

$$\text{ii. } \cos \theta = \frac{\text{base}}{\text{hypotenuse}} = \frac{B}{H}$$

$$\text{iii. } \tan \theta = \frac{\text{perpendicular}}{\text{base}} = \frac{P}{B}$$

$$\text{iv. } \operatorname{cosec} \theta = \frac{\text{hypotenuse}}{\text{perpendicular}} = \frac{H}{P}$$

$$\text{v. } \sec \theta = \frac{\text{hypotenuse}}{\text{base}} = \frac{H}{B}$$

$$\text{vi. } \cot \theta = \frac{\text{base}}{\text{perpendicular}} = \frac{B}{P}$$

2. Trigonometric Identities:

$$\text{i. } \sin^2 \theta + \cos^2 \theta = 1$$

$$\text{ii. } \sec^2 \theta - \tan^2 \theta = 1$$

$$\text{iii. } \operatorname{cosec}^2 \theta - \cot^2 \theta = 1$$

3. The numerical value of Trigonometric Ratios for some Standard Angles:

Angle θ $\rightarrow$	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not Defined
$\operatorname{cosec}\theta$	Not Defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not Defined
$\cot\theta$	Not Defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

4. Allied Angles Formulae Table:

Angles $\downarrow$	$\sin$	$\cos$	$\tan$	cosec	$\sec$	$\cot$
$-\theta$	$-\sin\theta$	$\cos\theta$	$-\tan\theta$	$-\operatorname{cosec}\theta$	$\sec\theta$	$-\cot\theta$
$\left(\frac{\pi}{2} - \theta\right)$ or $(90^\circ - \theta)$	$\cos\theta$	$\sin\theta$	$\cot\theta$	$\sec\theta$	$\operatorname{cosec}\theta$	$\tan\theta$
$\left(\frac{\pi}{2} + \theta\right)$ or $(90^\circ + \theta)$	$\cos\theta$	$-\sin\theta$	$-\cot\theta$	$\sec\theta$	$-\operatorname{cosec}\theta$	$-\tan\theta$
$(\pi - \theta)$ or $(180^\circ - \theta)$	$\sin\theta$	$-\cos\theta$	$-\tan\theta$	$\operatorname{cosec}\theta$	$-\sec\theta$	$-\cot\theta$
$(\pi + \theta)$ or $(180^\circ + \theta)$	$-\sin\theta$	$-\cos\theta$	$\tan\theta$	$-\operatorname{cosec}\theta$	$-\sec\theta$	$\cot\theta$
$\left(\frac{3\pi}{2} - \theta\right)$ or $(270^\circ - \theta)$	$-\cos\theta$	$-\sin\theta$	$\cot\theta$	$-\sec\theta$	$-\operatorname{cosec}\theta$	$\tan\theta$

$\left(\frac{3\pi}{2} + \theta\right) (270^\circ + \theta)$	$-\cos\theta$	$\sin\theta$	$-\cot\theta$	$-\operatorname{cosec}\theta$	$\operatorname{cosec}\theta$	$-\tan\theta$
$(2\pi - \theta) (360^\circ - \theta)$	$-\sin\theta$	$\cos\theta$	$-\tan\theta$	$-\operatorname{cosec}\theta$	$\sec\theta$	$-\cot\theta$
$(2\pi + \theta) (360^\circ + \theta)$	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\operatorname{cosec}\theta$	$\sec\theta$	$\cot\theta$
$(2n\pi + \theta) (360^\circ n + \theta)$	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\operatorname{cosec}\theta$	$\sec\theta$	$\cot\theta$

5. Sum and Difference Formulae:

- $\sin(x + y) = \sin x \cos y + \sin y \cos x$
- $\sin(x - y) = \sin x \cos y - \sin y \cos x$
- $\cos(x + y) = \cos x \cos y - \sin x \sin y$
- $\cos(x - y) = \cos x \cos y + \sin x \sin y$
- $\tan(x + y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$
- $\tan(x - y) = \frac{\tan x - \tan y}{1 + \tan x \tan y}$

6. Sum to Product Formulae:

- $\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$
- $\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$
- $\cos C + \cos D = 2\cos\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$
- $\cos C - \cos D = 2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{D-C}{2}\right)$

7. Product to Sum Formulae:

- $2\sin A \cos B = \sin(A + B) + \sin(A - B)$
- $2\cos A \sin B = \sin(A + B) - \sin(A - B)$
- $2\cos A \cos B = \cos(A + B) + \cos(A - B)$
- $2\sin A \sin B = \cos(A - B) - \cos(A + B)$

8. Multiple Angles Formulae:

- i. $\sin 2x = 2\sin x \cos x$
- ii. $\cos 2x = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$
- iii. $\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$
- iv. $\sin 3x = 3\sin x - 4\sin^3 x$
- v. $\cos 3x = 4\cos^3 x - 3\cos x$
- vi. $\tan 3x = \frac{3\tan x - \tan^3 x}{1 - 3\tan^2 x}$

9. Some Special Formulae:

- i. $\sin x \cdot \sin(60^\circ - x) \cdot \sin(60^\circ + x) = \frac{1}{4} \sin 3x$
- ii. $\cos x \cdot \cos(60^\circ - x) \cdot \cos(60^\circ + x) = \frac{1}{4} \cos 3x$
- iii. $\tan x \cdot \tan(60^\circ - x) \cdot \tan(60^\circ + x) = \tan 3x$

10. Maximum and Minimum of trigonometric functions:

- i. $a \sin^2 \theta + b \cos^2 \theta$

Maximum value = maximum between a and b .

Minimum value = minimum between a and b .

- ii. $\sin^n \theta \cdot \cos^n \theta$

If n is odd: Maximum value = $\frac{1}{2^n}$ and Minimum value = $-\frac{1}{2^n}$

If n is even: Maximum value = $\frac{1}{2^n}$ and Minimum value = 0.

- iii. $a \sin \theta + b \cos \theta$

Maximum value = $\sqrt{a^2 + b^2}$

Minimum value = $-\sqrt{a^2 + b^2}$

- iv. $a \sin^2 \theta + b \sec^2 \theta$

Minimum value = $2\sqrt{ab}$

- v. $a \cos^2 \theta + b \sec^2 \theta$

Minimum value = $2\sqrt{ab}$

- vi. $a \tan^2 \theta + b \cot^2 \theta$

Minimum value = $2\sqrt{ab}$

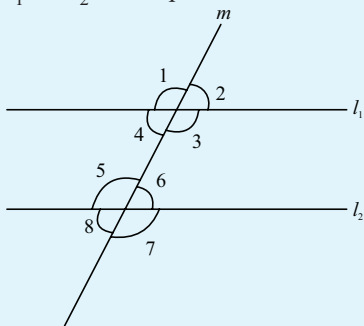
- vii. $a \operatorname{cosec}^2 \theta + b \sec^2 \theta$

Minimum value = $(\sqrt{a} + \sqrt{b})^2$

Geometry

1. Lines and Angles:

- i. Let l_1 and l_2 be two parallel lines and m be the transversal lines. Then,

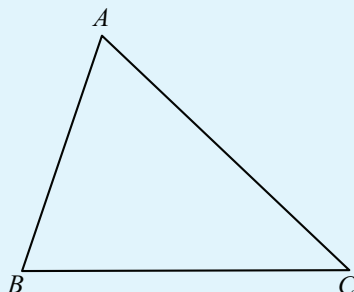


- a. Corresponding angles are equal
 $\angle 1 = \angle 5$, $\angle 2 = \angle 6$, $\angle 4 = \angle 8$, $\angle 3 = \angle 7$
- b. Alternate Angles are equal
 $\angle 4 = \angle 6$, $\angle 3 = \angle 5$
- c. Vertically Opposite angles are equal
 $\angle 1 = \angle 3$, $\angle 2 = \angle 4$, $\angle 5 = \angle 7$, $\angle 6 = \angle 8$
- d. Alternate exterior angles are equal
 $\angle 1 = \angle 7$, $\angle 2 = \angle 8$
- e. Sum of co-interior angles are equal
 $\angle 4 + \angle 5 = 180^\circ$, $\angle 3 + \angle 6 = 180^\circ$
- f. Sum of co-interior angles are equal
 $\angle 1 + \angle 8 = 180^\circ$, $\angle 2 + \angle 7 = 180^\circ$

- ii. The complementary angle of $\theta = 90^\circ - \theta$.
- iii. The supplementary angle of $\theta = 180^\circ - \theta$.

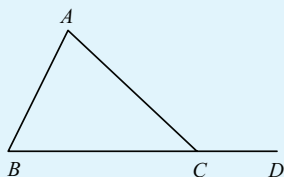
2. Triangle:

- i. The sum of all the angles of a triangle is 180° .
i.e. $\angle BAC + \angle ABC + \angle ACB = 180^\circ$



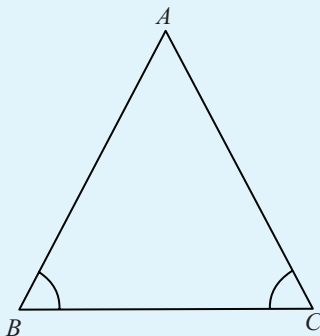
- ii. The exterior angle of a triangle equals the sum of opposite interior angles

i.e. $\angle ACD = \angle ABC + \angle BAC$



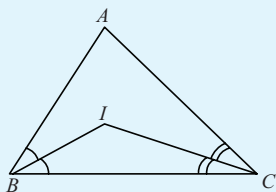
- iii. The angles opposite to the equal sides of a triangle are equal.

i.e. $AB = AC \Leftrightarrow \angle ACB = \angle ABC$



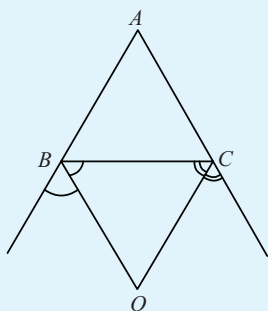
- iv. Let IB and IC be the internal angles bisectors of a triangle ABC .
Then,

$$\angle BIC = 90^\circ + \frac{\angle BAC}{2}$$

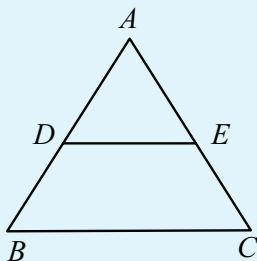


- v. Let OB and OC be the external angles bisectors of a triangle ABC .

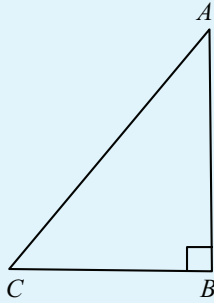
$$\text{Then, } \angle BOC = 90^\circ - \frac{\angle BAC}{2}$$



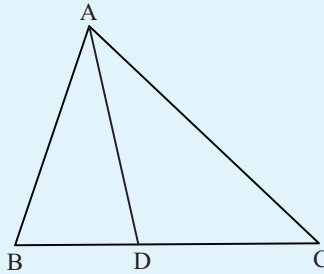
- vi. **Thales Theorem:** If D and E are two points on sides A and B of $\triangle ABC$ such that $DE \parallel BC$, then $\frac{AD}{DB} = \frac{AE}{EC} = \frac{AB}{AC}$



- vii. **Mid Point Theorem:** If D and E are the mid points of sides AB and AC respectively of $\triangle ABC$, then $DE \parallel BC$ and $DE = \frac{BC}{2}$.
The converse of this theorem is also true.
- viii. **Pythagoras Theorem:** Let ABC be a right angled triangle right angle at B . Then, $AB^2 + BC^2 = AC^2$

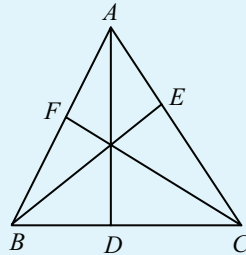


- ix. **Interior Angle Bisector Theorem:** Let AD be the angle bisector of $\angle BAC$, then $\frac{AB}{AC} = \frac{BD}{CD}$

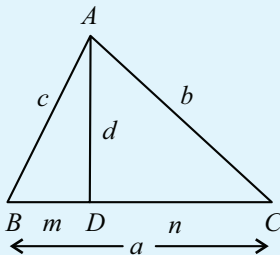


- x. **Ceva's Theorem:** Let AD , BE , and CF be the cevians of $\triangle ABC$. Then,

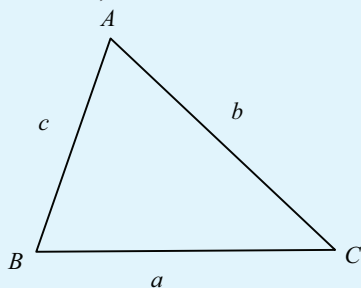
$$\frac{AE}{CE} \cdot \frac{CD}{BD} \cdot \frac{BF}{AF} = 1$$



- xi. **Stewart's Theorem:** Let AD be a cevian of length d dividing BC into segments $BD = m$ and $CD = n$. Then, $b^2m + c^2n = a(d^2 + mn)$

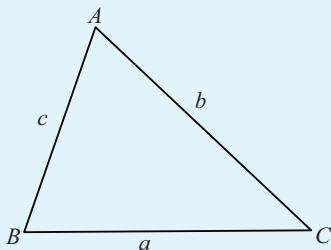


xii. **Sine Rule:** In $\triangle ABC$, we have:



$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R \text{ Where R is the circumradius of the triangle.}$$

xiii. **Cosine rule:** In $\triangle ABC$, we have:



$$\cos A = \frac{b^2 + c^2 - a^2}{2bc} \quad \cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

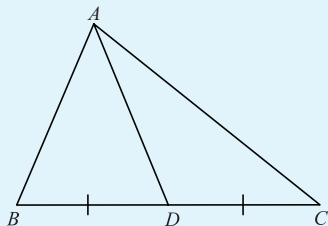
$$\cos C = \frac{b^2 + a^2 - c^2}{2ba}$$

xiv. **Exterior Angle Bisector Theorem:** Let AD be the angle bisector of

$$\triangle BAC, \text{ then } \frac{AB}{AC} = \frac{BD}{CD}$$

xv. **Apollonius Theorem:** Let AD be a median of $\triangle ABC$, then

$$AB^2 + AC^2 = 2(AD^2 + BD^2)$$



xvi. In any right angle triangle, we have:

$$a^2 + b^2 + c^2 = 8R^2$$

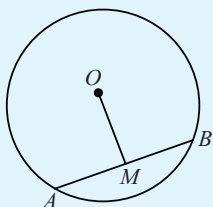
where a , b and c are sides of triangle and R is the circumradius.

3. Centroid, Orthocentre, Incentre, and Circumcentre:

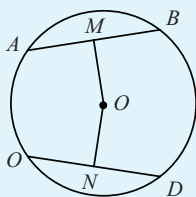
- i. Orthocentre, Centroid, and Circumcentre lie on a straight line and the centroid divides the line segment joining the orthocentre and circumcentre in the ratio 2 : 1.
- ii. The median divides the triangle into two parts of equal areas.
- iii. The Circumcentre of a right-angle triangle lies on the mid-point of the hypotenuse.
- iv. The orthocentre of an obtuse angle triangle lies outside the triangle.
- v. The orthocentre of a right angle triangle lies on the vertex of the triangle where 90° angle is formed

4. Circle:

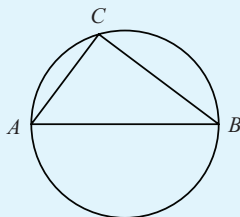
- i. The perpendicular from the centre of the circle to the chord bisects the chord, If $OM \perp AB$ then $AM = BM$



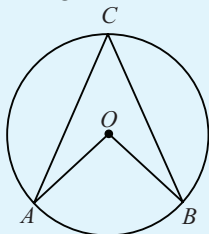
- ii. If AB and CD are two chords of a circle with centre O such that $OM \perp AB$ and $ON \perp CD$, then $OM = ON \Leftrightarrow AB = CD$.



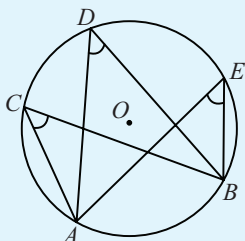
- iii. The angle contained in a semi-circle is equal to 90° i.e. if AB is a diameter of the circle and C is any point on the circumference of the circle then $\angle ACB = 90^\circ$



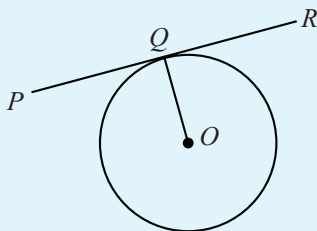
- iv. The angle subtends by an arc at the centre of the circle is twice the angle subtended by the same arc at the remaining part of the circle,
 $\angle AOB = 2\angle ACB$.



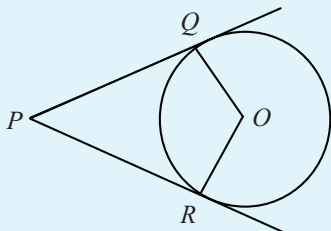
- v. Angles subtend by an arc on the remaining part of the circle are equal
 i.e. $\angle ACB = \angle ADB = \angle AEB$.



- vi. The radius through the point of contact is perpendicular to the tangent at that point, if PQR is a tangent to the circle with centre O , then
 $\angle PQO = 90^\circ$



- vii. If PQ and PR are two tangents to the circle with centre O drawn from an external point P , then

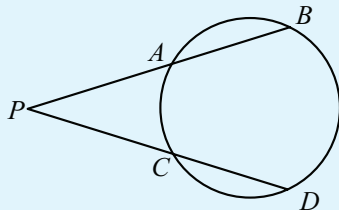


a. $\angle RPQ + \angle ROQ = 180^\circ$

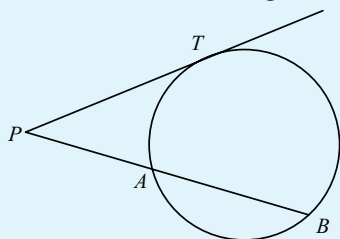
b. $\angle OQP = \angle ORP$

c. $\angle POQ = \angle POR$

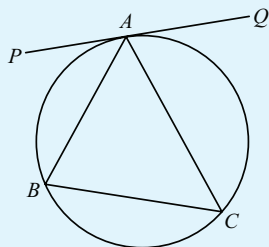
viii. If PAB and PCD are two secants of a circle then $PA \times PB = PC \times PD$



ix. If PAB is a secant and PT is a tangent to a circle then $PA \times PB = PT^2$

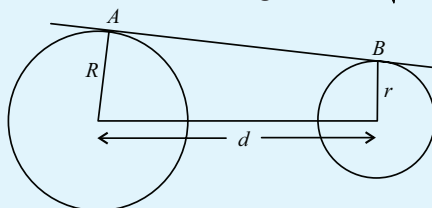


x. Let A, B and C be three points on a circle and PAQ be the tangent at point A , then $\angle PAB = \angle ACB$ and $\angle QAC = \angle ABC$

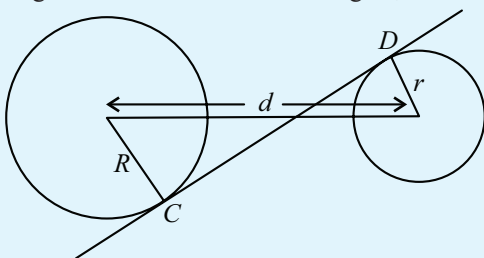


xi. Let the radii of two circles be R and r and let the distance between their centres be d .

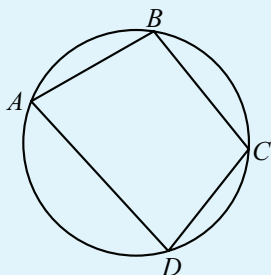
a. The length of direct common tangent, $AB = \sqrt{d^2 - (R - r)^2}$



- b. The length of transverse common tangent, $CD = \sqrt{d^2 - (R+r)^2}$



- xi. The sum of opposite interior angles of a cyclic quadrilateral is 180°



- xii. If two chords AB and CD of the circle intersect each other perpendicularly at O then, Radius of the circle

$$= \frac{\sqrt{AO^2 + BO^2 + CO^2 + DO^2}}{2}$$

5. Inradius and Circumradius:

- i. If a is the length of the side of an equilateral triangle, then its

Circumradius, $R = \frac{a}{\sqrt{3}}$

- ii. If a is the length of the side of an equilateral triangle, **then its Inradius**,

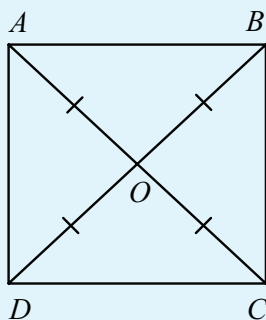
$$r = \frac{a}{2\sqrt{3}}$$

- iii. If the sides of a right angle triangle are a , b , and c , where c is the hypotenuse, then its **Circumradius**, $R = \frac{c}{2}$

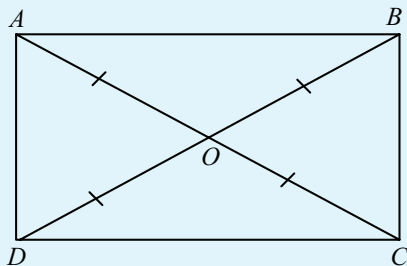
- iv. If the sides of a right angle triangle are a , b and c , where c is the hypotenuse, then its **Inradius**, $r = \frac{a+b-c}{2}$

- v. If the sides of a triangle are a, b and c , then its **Circumradius**, $R = \frac{abc}{4\Delta}$, where Δ is the area of the triangle.
- vi. If the sides of a triangle are a, b , and c , then its **Inradius**, $r = \frac{\Delta}{s}$, where Δ is the area of the triangle and s is the semi perimeter of the triangle.
- vii. Distance between incenter and circumcenter $= \sqrt{R^2 - 2rR}$ Where R is the circumradius and r is the inradius.

6. **Square:** Let $ABCD$ be a square and let the diagonals AC and BD intersect at point O . Then,



- $AC = BD$
 - $OA = OB = OC = OD$
 - $\angle AOB = \angle BOC = \angle COD = \angle DOA = 90^\circ$
 - $\angle BAC = 45^\circ$
7. **Rectangle:** Let $ABCD$ be a rectangle and let the diagonals AC and BD intersect each other at point O . Then,



- $AC = BD$
- $OA = OB = OC = OD$

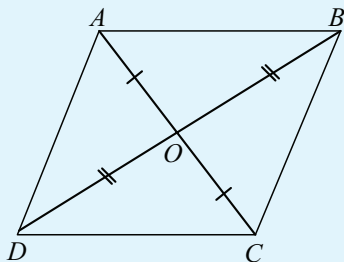
- 8. Parallelogram:** Let $ABCD$ be a parallelogram and let the diagonals AC and BD intersect each other at point O . Then

$$AO = CO \text{ and } BO = DO$$

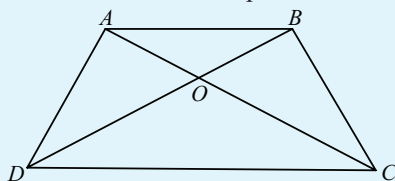
- In parallelogram $ABCD$, X and Y are the lengths of the two diagonal then,

$$X^2 + Y^2 = AB^2 + BC^2 + CD^2 + DA^2$$

- 9. Rhombus:** Let $ABCD$ be a rhombus and let the diagonals AC and BD intersect each other at point O . Then,



- $AB = BC = CD = DA$
 - $\angle AOB = \angle BOC = \angle COD = \angle DOA = 90^\circ$
 - $OA = OC$ and $OB = OD$
 - $AC^2 + BD^2 = 4AB^2$
- 10. Trapezium:** Let $ABCD$ be a trapezium with $AB \parallel CD$ and let the diagonals AC and BD intersect each other at point O . Then,



- $\triangle AOB \sim \triangle COD$
- $OA \times OD = OC \times OB$
- If a and b are the parallel sides of the trapezium and P and Q are mid points of unparallel sides then $PQ = \frac{a+b}{2}$
- If a and b are the parallel sides of the trapezium and P and Q are mid points of diagonals then $PQ = \frac{a-b}{2}$

- v. Sum of squares of diagonal $= AC^2 + BD^2 + 2AB \times CD$
- vi. Any isosceles trapezium is a cyclic quadrilateral.
- vii. Any trapezium inscribed in a circle is always isosceles trapezium.

11. Polygon

- i. The sum of all interior angles of a polygon of n sides $= (n - 2) \times 180^\circ$
- ii. The measure of each interior angle of a regular polygon of n sides

$$= \frac{(n - 2) \times 180^\circ}{n}$$
- iii. The sum of all exterior angles of a polygon of n sides $= 360^\circ$
- iv. The measure of each exterior angle of a regular polygon of n sides

$$= \frac{360^\circ}{n}$$
- v. Number of diagonals in a polygon of n sides $= \frac{n(n - 3)}{2}$

Average

1. $\text{Average} = \frac{\text{Sum of observations}}{\text{Number of observations}}$
2. $\text{Sum of numbers} = \text{Average} \times \text{number of observations}$
3. If the average of n_1 numbers is a_1 and the average of n_2 numbers is a_2 ,
then the combined average of all the numbers $= \frac{a_1 n_1 + a_2 n_2}{n_1 + n_2}$
4. Let x_i be n observations and w_i be their respective weights, then the
weighted average $= \frac{x_1 w_1 + x_2 w_2 + \dots + x_n w_n}{w_1 + w_2 + \dots + w_n}$
5. **Average of numbers:**
 - i. The average of the first n natural numbers $= n(n+1)$
 - ii. The average of the first n even natural numbers $= (n+1)$
 - iii. The average of the first n odd natural numbers $= n$
 - iv. The average of squares of the first n natural numbers $= \frac{(n+1)(2n+1)}{6}$
 - v. The average of squares of the first n even natural numbers
 $= \frac{2(n+1)(2n+1)}{3}$
 - vi. The average of squares of the first n odd natural numbers
 $= \frac{(2n-1)(2n+1)}{3}$
 - vii. The sum of cubes of the first n natural numbers $= n \left[\frac{(n+1)}{2} \right]^2$

- viii. The sum of cubes of the first n even natural numbers $= 2n[(n+1)]^2$
- ix. The sum of cubes of the first n odd natural numbers $= n(2n^2 - 1)$
- x. Let a be the first term and l be the last term of an Arithmetic Progression (AP). Then, the average of all the numbers $= \frac{a+l}{2}$

Ratio and Proportion

1. **Ratio:** The ratio of two numbers a and b is written as $a : b$ and is equivalent to $\frac{a}{b}$ i.e. $a : b = \frac{a}{b}$
2. **Proportion:**
 - i. If a, b, c , and d are in proportion then we write $a : b :: c : d$
 $a : b :: c : d = a \times d = b \times c$
 - ii. **Continued Proportion:** If a, b and c are in continued proportion, then $\frac{a}{b} = \frac{b}{c}$
 - iii. **Third proportion:** The third proportion of a and $b = \sqrt{ab}$
 - iv. **Fourth proportion:** The fourth proportion of a, b , and $c = \frac{bc}{a}$
 - v. **Mean proportion:** The mean proportion of a and $b = \sqrt{ab}$
3. **Directly and Indirectly Proportional:**
 - i. **Directly Proportional:** If X is directly proportional to Y , then we write $X \propto Y$,
 $X \propto Y \Rightarrow X = kY$, where k is a constant
 Also, $X \propto Y \Rightarrow \frac{X_1}{X_2} = \frac{Y_1}{Y_2}$
 - ii. **Indirectly Proportional:** If X is indirectly proportional to Y , then we write $X \propto \frac{1}{Y}$.

$$X \propto \frac{1}{Y} \Rightarrow XY = k, \text{ where } k \text{ is a constant}$$

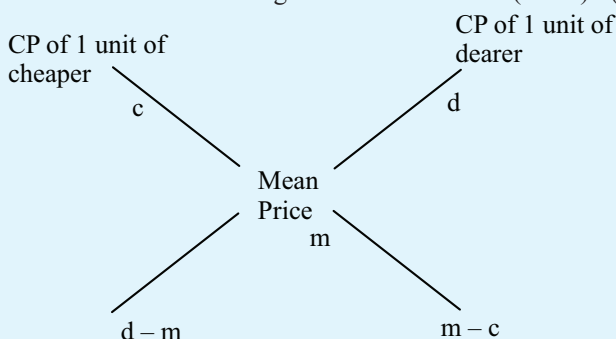
$$\text{Also, } X \propto \frac{1}{Y} \Rightarrow X_1 Y_1 = X_2 Y_2$$

4. Partnership:

- If A invests ₹ x for t_1 time and B invests ₹ y for t_2 time, then the ratio of their profits = $(xt_1) : (yt_2)$
- If A invests ₹ x for t_1 time, B invests ₹ y for t_2 time and C invests ₹ z for t_3 then the ratio of their profits = $(xt_1) : (yt_2) : (zt_3)$

5. Mixture and Alligation:

- A container contains x litres of milk. From this container y litres of milk was taken out and replaced by water. If this process was repeated n times, then the quantity of milk in the final mixture = $x \left(1 - \frac{y}{x}\right)^n$
- The cost price of 1 unit of cheaper ingredient is c and the cost price of 1 unit of dearer ingredient is d . If these two ingredients are mixed together such that the cost price of 1 unit of mixture is m , then the ratio in which these two ingredients are mixed = $(d - m) : (m - c)$



$$\text{Ratio of Quantity of cheaper to dearer} = (d - m) : (m - c)$$

1. Conversion of percentage into fraction and vice versa

i. $x\% = \frac{x}{100}$

ii. $\frac{x}{y} = \left(\frac{x}{y} \times 100 \right)\%$

2. If A is $x\%$ more than B , then $A = (100 + x)\%$

3. If A is $y\%$ less than B , then $A = (100 - y)\%$

4. If A is $x\%$ more than B , then B is $\frac{x}{100+x}\%$ less than A

5. If A is $y\%$ less than B , then B is $\frac{y}{100-y}\%$ more than A

6. If A is $x\%$ more than B and B is $y\%$ more than C , then A is $\left(x + y + \frac{xy}{100} \right)\%$ more than C .

7. If the price of a commodity increased by $x\%$, then the reduction in consumption so that the expenditure remains same = $\frac{x}{100+x}\%$

8. Let the population of a town increases $R\%$ every year. If the present population of a town is P , then

i. n years ago, the population of the town n years ago = $\frac{P}{\left(1 + \frac{R}{100} \right)^n}$

ii. n years hence, the population of the town = $P \left(1 + \frac{R}{100} \right)^n$

9. Let the value of an item depreciates $R\%$ every year. If the present value of the item is P , then

i. n years ago, the value of the item $= \frac{P}{\left(1 + \frac{R}{100}\right)^n}$

ii. n years hence, the value of the item $= P\left(1 + \frac{R}{100}\right)^n$

Profit and Loss

1. Some basic formulae:

- i. When $SP > CP$, then profit = $SP - CP$
- ii. When $CP > SP$, then loss = $CP - SP$
- iii. $SP = CP + \text{Profit}$
- iv. $SP = CP - \text{Loss}$

$$\text{v. Profit \%} = \frac{SP - CP}{CP} \times 100 = \frac{\text{Profit}}{CP} \times 100$$

$$\text{vi. Loss \%} = \frac{CP - SP}{CP} \times 100 = \frac{\text{Loss}}{CP} \times 100$$

2. If profit % = P , then

$$\text{i. } SP = \frac{100 + P}{100} \times CP$$

$$\text{ii. } CP = \frac{100}{100 + P} \times SP$$

3. If loss % = L , then

$$SP = \frac{100 - L}{100} \times CP$$

$$CP = \frac{100}{100 - L} \times SP$$

- 4.** A person sells two articles at the same price. If he incurs $x\%$ loss on one item and $x\%$ profit on another item, then he incurs a loss of $\frac{x^2}{100}\%$ on the whole transaction.

5. If a shopkeeper sells his goods at cost price but uses false weight, then his profit % = $\frac{\text{True weight} - \text{false weight}}{\text{False weight}} \times 100$
6. If a shopkeeper sells his goods at $x\%$ profit and uses $y\%$ less weight, then his profit % = $\frac{x + y}{100 - y}$
7. If a shopkeeper sells his goods at $x\%$ loss and uses $y\%$ less weight, then his profit % or loss % = $\frac{y - x}{100 - y}$

Discount

1. Discount = MP – SP

2. Discount % = $\frac{\text{Discount}}{MP} \times 100 = \frac{MP - SP}{MP} \times 100$

3. If discount % = d , then

i. $SP = \frac{100 - d}{100} \times MP$

ii. $MP = \frac{100}{100 - d} \times SP$

4. When there are two successive discounts of $d_1\%$ and $d_2\%$, then

$$SP = \frac{(100 - d_1)}{100} \times \frac{(100 - d_2)}{100} \times MP$$

5. A single discount equivalent to two successive discounts of $x\%$ and $y\%$

$$= \left(x + y - \frac{xy}{100} \right) \%$$

6. If there is a profit of $P\%$ after giving $D\%$ discount, then $\frac{MP}{CP} = \frac{100 + P}{100 - D}$

Simple Interest and Compound Interest

1. If Principal = P , Rate of interest = $R\%$ per annum and Time = T years. Then,

i. Simple Interest, $SI = \frac{P \times R \times T}{100}$

ii. Amount, $A = P + SI$

2. If an amount of P at a certain rate of interest becomes n_1 times in t years, then it will become n_2 times in $= \frac{(n_2 - 1) \times t}{n_1}$

3. If the Principal = P , Rate of interest = $R\%$ per annum and Time = T years and the interest is compounded annually, then,

i. Amount, $A = P \left(1 + \frac{R}{100} \right)^T$

ii. Compound Interest, $CI = A - P = P \left[\left(1 + \frac{R}{100} \right)^T - 1 \right]$

4. If the Principal = P , Rate of interest = $R\%$ per annum and Time = T years and the interest is compounded half-yearly, then,

i. Amount, $A = P \left(1 + \frac{R}{2 \times 100} \right)^{2T}$

ii. Compound Interest, $CI = A - P = P \left[\left(1 + \frac{R}{2 \times 100} \right)^{2T} - 1 \right]$

5. If the Principal = P , Rate of interest = $R\%$ per annum and Time = T years and the interest is compounded quarterly, then,

i. Amount, $A = P \left(1 + \frac{R}{4 \times 100} \right)^{4T}$

ii. Compound Interest, $CI = A - P = P \left[\left(1 + \frac{R}{4 \times 100} \right)^{4T} - 1 \right]$

6. If the interest is compounded annually and the rate of interest is $R_1\%$ for the first year, $R_2\%$ for the second year and $R_3\%$ for the third year, then

$$A = P \left(1 + \frac{R_1}{100} \right) \left(1 + \frac{R_2}{100} \right) \left(1 + \frac{R_3}{100} \right)$$

7. If the interest is compounded annually, the rate of interest is $R\%$ per annum and the time is in fractions say $t \frac{n}{m}$ years, then

$$A = P \left(1 + \frac{R}{100} \right)^t \left(1 + \frac{nR}{m \times 100} \right)$$

8. If an amount P at a certain rate of interest becomes n_1 times in t_1 years, and n_2 times in t_2 years, and the interest is compounded annually, then $(n_2)^{t_1} = (n_1)^{t_2}$

9. If Principal = P , Rate of interest = $R\%$ per annum and Time = 2 years. Then,

i. $CI - SI = P \left(\frac{R}{100} \right)^2$

ii. $CI - SI = \frac{SI \times R}{100}$

10. If Principal = P , Rate of interest = $R\%$ per annum and Time = 3 years.

Then, $CI - SI = P \left[\left(\frac{R}{100} \right)^2 + 3 \left(\frac{R}{100} \right)^2 \right]$

11. If an amount of P is to be returned in n equal installments and the value of each installment is I , then

$$P = \frac{I}{\left(1 + \frac{R}{100} \right)} + \frac{I}{\left(1 + \frac{R}{100} \right)^2} + \frac{I}{\left(1 + \frac{R}{100} \right)^3} + \dots + \frac{I}{\left(1 + \frac{R}{100} \right)^n}$$

Time and Distance

1. Basic Formulae:

i. $\text{Distance} = \text{Speed} \times \text{Time}$

ii. $\text{Time} = \frac{\text{Distance}}{\text{Speed}}$

iii. $\text{Speed} = \frac{\text{Distance}}{\text{Time}}$

2. Conversion of km/hr to m/s and vice-versa

i. $x \text{ km/hr} = x \times \frac{5}{18} \text{ m/s}$

ii. $x \text{ m/s} = x \times \frac{18}{5} \text{ km/hr}$

3. Average Speed:

i. $\text{Average Speed} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$

ii. If a person covers a certain distance at x kmph and the same distance at y kmph then the average speed during the whole journey

$$= \frac{2xy}{x+y} \text{ kmph.}$$

4. Same Distance is covered at two Different Speeds:

i. If a person covers a distance from A to B at x kmph and returns back to A at y kmph and the total time of the journey is t hours then the distance between A and B $= \left(\frac{xy}{x+y} \times t \right)$ kmph.

- ii. If a person covers the same distance at two different speeds x kmph and y kmph and the difference in time taken is t hours then the distance between A and B = $\left(\frac{xy}{x-y} \times t \right)$ kmph.

5. Relative Speed:

- i. When two persons are moving in the same direction at x kmph and y kmph (where $x > y$), then the relative speed of A with respect to B = $(x - y)$ kmph
- ii. When two persons A and B are moving in the opposite direction at x kmph and y kmph, then the relative speed of A with respect to B = $(x + y)$ kmph

6. If two trains A and B start at the same time towards each other from two fixed points and after passing each other they complete their journey in X and Y hours respectively, then

$$\frac{\text{Speed of A}}{\text{Speed of B}} = \frac{\sqrt{X}}{\sqrt{Y}}$$

7. Trains:

- i. If the length of the train = L m and the speed of the train = S m/s, then the time taken (in seconds) by the train to cross a pole, tree etc. is

$$= \frac{L}{S}$$

- ii. If the length of the train = L_1 m, the speed of the train = S m/s, and the length of the platform or bridge = L_2 m, then the time taken (in seconds) by the train the platform is $t = \frac{L_1 + L_2}{S}$

- iii. If the length of the train = L m, the speed of the train = S_1 m/s, and the speed of the man = S_2 m/s, then the time taken (in seconds) by the train to cross the man moving in the same direction as of the train is

$$t = \frac{L}{S_1 - S_2}$$

- iv. If the length of the train = L m, the speed of the train = S_1 m/s, and the speed of the man = S_2 m/s, then the time taken (in seconds) by the train to cross the man moving in the opposite direction as of the

$$\text{train is } t = \frac{L}{S_1 + S_2}$$

- v. If the length of the train A = L_1 m, the speed of the train A = S_1 m/s, the length of the train B = L_2 m, and the speed of the train B = S_2 m/s, and the trains are running on parallel tracks in the same direction, then the time taken (in seconds) by the faster train to cross the slower

train is $t = \frac{L_1 + L_2}{S_1 - S_2}$

- vi. If the length of the train A = L_1 m, the speed of the train A = S_1 m/s, the length of the train B = L_2 m, and the speed of the train B = S_2 m/s, and the trains are running on parallel tracks in the opposite direction, then the time taken (in seconds) by the trains to cross each other is

$$t = \frac{L_1 + L_2}{S_1 + S_2}$$

Boats and Streams

1. If the speed of a boat in still water is x kmph and the speed of the stream is y kmph, then
 - i. Speed downstream = $(x + y)$ kmph
 - ii. Speed upstream = $(x - y)$ kmph
2. If the speed downstream is x kmph and the speed upstream is y kmph, then
 - i. Speed of the boat in still water = $\frac{1}{2}(x + y)$ kmph
 - ii. Speed of the stream = $\frac{1}{2}(x - y)$ kmph
3. If the speed of the boat in still water is x kmph and the speed of the stream is y kmph and it takes t hours to row from A to B and come back, then the distance between A and B = $\frac{t(x^2 - y^2)}{2x}$
4. If the speed of a boat in still water is x kmph and the speed of the stream is y kmph, let the boat covers the same distance upstream and downstream, then the ratio of time taken downstream and upstream = $(x - y) : (x + y)$.
5. If the speed of a boat in still water is x kmph and the speed of the stream is y kmph, let the boat takes n times as long to row up as to row down, then $x = y\left(\frac{n+1}{n-1}\right)$

Time and Work

1. If A can do a work in m days and B can do the same work in n days, then A and B together can do the same work in $= \frac{mn}{m+n}$ days
2. If A can do a work in m days and A and B together can do the same work in n days, then B can do the same work in $= \frac{mn}{m-n}$ days
3. If A takes x days more than A and B together to do a work and B takes y days more than A and B together to do a work then, A and B together complete the work in $= \sqrt{xy}$ days.
4. If M_1 men can do W_1 work in D_1 days working H_1 hours per day and M_2 men can do W_2 work in D_2 days working H_2 hours per day then,
$$\frac{M_1 D_1 H_1}{W_1} = \frac{M_2 D_2 H_2}{W_2}$$
5. A and B together complete a work. If A can do the same work in m days and B can do the same work in n days, then the ratio of their wages $= n : m$